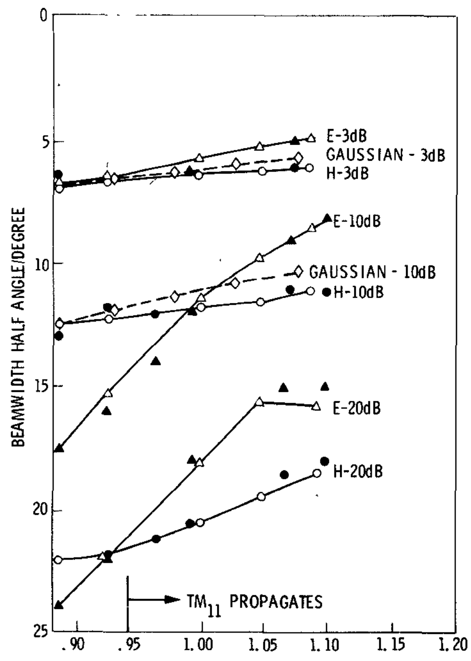

Fig. 3. Frequency variation of 3, 10, and 20-dB points on the far-field patterns. Frequency is in units of the center frequency, and angles are in degrees. Solid figures represent data measured near $\lambda_0 = 34.2$ mm and open figures represent data measured near $\lambda_0 = 1.4$ mm.

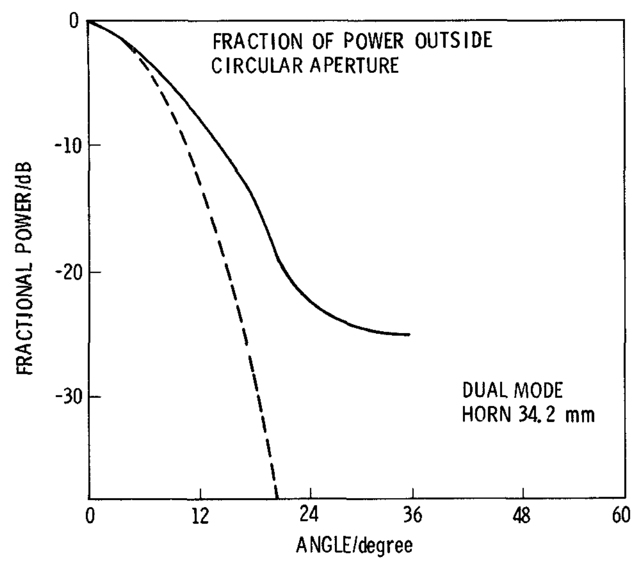

Fig. 4. Fractional power outside a cone of a given polar angle.

levels are considerably reduced in comparison with single-mode radiation patterns for a cylindrical waveguide [3].

The beam efficiency was obtained by integrating the 34.2-mm patterns over all solid angles. Fig. 4 shows the fractional power outside a cone of a given polar angle.

For the 34.2-mm horn, far-field phase measurements were performed. The phase center was found to be 6.46 λ_0 back from the aperture. The far-field phase deviation relative to a spherical wave centered at this point are summarized in Fig. 5.

Waveguide reflection loss was measured over a 30-percent bandwidth centered at 34.2 mm. The reflected power was less

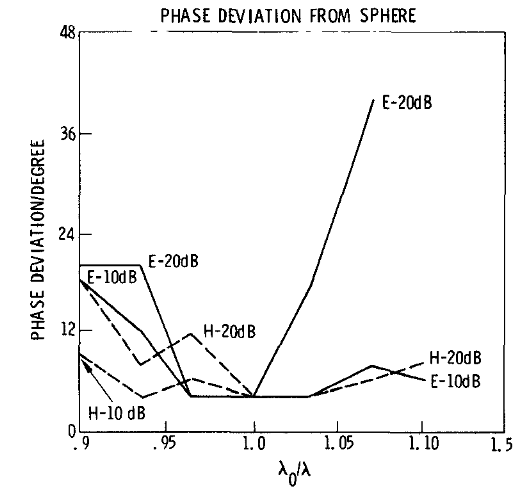

Fig. 5. Far-field phase deviation in degrees from a sphere centered at the phase center measured near $\lambda_0 = 34.2$ mm.

than -23 dB over the whole band, corresponding to a VSWR of 1.15.

IV. Conclusions

We have designed and tested a dual-mode horn at four different wavelengths. The horn has low sidelobe levels and nearly equal E and H plane patterns. The patterns provide a good match to a Gaussian beam. The far-infrared patterns have a single well-defined phase center which is independent of polarization and frequency. The VSWR is quite low, and adequate for near-millimeter applications. The bandwidth of the horn is between 10 and 20 percent, depending on the level of acceptable performance required.

Acknowledgment

We would like to thank H. F. Reilly, Jr. for assistance in making the 34.2-mm measurements.

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An Analytical Method for the Capacitance of the Rectangular Inhomogeneous Coaxial Line Having Anisotropic Dielectrics

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***Abstract*—An analytical technique for the capacitance of a rectangular inhomogeneous coaxial line with zero thickness offset inner conductor and**

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having anisotropic dielectrics is briefly outlined. The spectral domain technique in discrete Fourier variable, under quasi-static approximation, is used to determine the capacitance expression. Numerical data are presented on the capacitance of rectangular homogeneous/inhomogeneous coaxial lines having anisotropic dielectrics. The capacitance values obtained using the analytical technique presented are found to be in good agreement with the results reported using numerical techniques by other authors.

I. Introduction

Rectangular coaxial transmission lines consisting of an air dielectric with a thin center conductor have been extensively used in electromagnetic (EM) susceptibility and emission testing, calibration of radiation survey meters, biological effects of RF exposure, and electric and magnetic field probes for VHF and UHF bands [1]–[2]. The impedance data on this transmission line obtained by various workers, using different analytical and numerical techniques, are reviewed and compared in [1]. The data on the capacitances of rectangular structures for the edge-offset and also broadside-offset finite-thickness strip conductor have been obtained by Chen [3]. Singular integral equation technique has been used by Tippet and Chang [4] to derive the capacitance formula of the rectangular coaxial line with offset zero thickness inner conductor. A simpler method known as the 'Transverse Transmission Line' method in conjunction with the variational technique in space domain has been reported by Bhat and Koul [5] for analyzing edge-offset and broadside-offset shielded stripline, also known as rectangular coaxial line, with isotropic dielectrics. In subsequent papers, this technique has been extended to analyze shielded structures with uniaxially anisotropic substrates [6]–[7].

Recently, a numerical calculation of the capacitance of a rectangular homogeneous coaxial line with offset inner conductor having an anisotropic dielectric with tilted optical axis has been reported [8]. Data on capacitance of this transmission line for various angles of tilt between the principal axes of the substrate and the $x-y$ coordinate system are given in this paper. To date, there is no analytical technique reported in the literature for solving such structures. Further, there is no analytical/numerical technique reported for analyzing the capacitance characteristics of a rectangular inhomogeneous coaxial transmission line having anisotropic dielectrics.

In this paper, a brief outline of the analytical technique used is described. The method uses the spectral domain technique with discrete Fourier variable, under quasi-static approximation. The capacitance expression for rectangular inhomogeneous coaxial lines with offset inner conductor having anisotropic dielectric substrates is presented. A comparison of the capacitance characteristics of rectangular symmetric homogeneous coaxial lines using the present theory with those reported in [8] is made. Numerical data are presented for a rectangular inhomogeneous coaxial line having anisotropic dielectrics.

II. Brief Outline of the Analytical Procedure

Consider a general rectangular coaxial line having an offset inner conductor sandwiched between three anisotropic dielectrics as shown in Fig. 1. The permittivity and the angle of tilt between the principal axes of the substrate and the $x-y$ coordinate system in the cross-sectional plane are indicated in the figure. The permittivity tensor $\hat{\epsilon}_i$ in the $x-y$ coordinate system can be obtained using the coordinate transformation [9]

$$\hat{\epsilon}_i(x,y)=\epsilon_0\begin{bmatrix}\epsilon_{xxi} & \epsilon_{xyi}\\ \epsilon_{xyi} & \epsilon_{yyi}\end{bmatrix}, \qquad i=1,2,3 \tag{1a}$$

where

$$\epsilon_{xxi}=\epsilon_{\xi i}\cos^2\theta_i+\epsilon_{\eta i}\sin^2\theta_i \tag{1b}$$

$$\epsilon_{xyi}=(\epsilon_{\eta i}-\epsilon_{\xi i})\sin\theta_i\cos\theta_i \tag{1c}$$

$$\epsilon_{yyi}=\epsilon_{\eta i}\cos^2\theta_i+\epsilon_{\xi i}\sin^2\theta_i. \tag{1d}$$

The capacitance of the rectangular coaxial line can be obtained by determining the quasi-static potential distribution function in the spectral domain. The static potential distribution satisfies the Laplace's equation

$$\nabla_t\cdot[\hat{\epsilon}_i\cdot\nabla_t\phi_i(x,y)]=0. \tag{2}$$

Taking the Fourier transform with respect to x, we get

$$\epsilon_{yyi}\frac{\partial^2\hat{\phi}_i(\beta_n,y)}{\partial y^2}+2j\epsilon_{xyi}\beta_n\frac{\partial\hat{\phi}_i(\beta_n,y)}{\partial y}-\beta_n^2\epsilon_{xxi}\hat{\phi}_i(\beta_n,y)=0 \tag{3}$$

where β_n is a discrete Fourier variable.

Assuming the strip conductor to be infinitesimally thin and the charge distribution to be represented as

$$\rho(x,y)=f(x)\delta\left(y-\frac{b}{2}-h_y\right) \tag{4}$$

where $\delta(y-b/2-h_y)$ is the Dirac's delta function.

Now writing down the solution of $\hat{\phi}(\beta_n,y)$ from (3) in various regions and matching the boundary conditions at various dielectric interfaces, we get

$$\hat{\phi}\left(\beta_n,\frac{b}{2}+h_y\right)=\frac{\hat{f}(\beta_n)}{\beta_n\epsilon_0 Y} \tag{5a}$$

where

$$Y=\left[\epsilon_{rf3}\coth\left(\beta_n\left(\frac{b}{2}-h_y\right)F_3\right)+\epsilon_{rf2}\left\{\left(\epsilon_{rf1}\coth\left(\frac{\beta_n bF_1}{2}\right)\cdot\coth(\beta_n h_yF_2)+\epsilon_{rf2}\right)\Big/\left(\epsilon_{rf2}\coth(\beta_n h_yF_2)+\epsilon_{rf1}\coth\left(\frac{\beta_n bF_1}{2}\right)\right)\right\}\right] \tag{5b}$$

$$\epsilon_{rfi}=\sqrt{\epsilon_{\xi i}\epsilon_{\eta i}} \tag{5c}$$

$$F_i=\left[\sqrt{\epsilon_{\eta i}/\epsilon_{\xi i}}\right]\left[1+\{\epsilon_{\eta i}/\epsilon_{\xi i}-1\}\cos^2\theta_i\right]^{-1} \tag{5d}$$

$$\hat{f}(\beta_n)=\int_{\text{strip}}f(x)\sin\beta_n x\,dx \tag{5e}$$

$$\beta_n=\frac{n\pi}{2c}, \qquad n=2,4,\cdots,\infty. \tag{5f}$$

The potential distribution function $\hat{\phi}(\beta_n,b/2+h_y)$ can be written as

$$\hat{\phi}\left(\beta_n,\frac{b}{2}+h_y\right)=\hat{\phi}_c\left(\beta_n,\frac{b}{2}+h_y\right)+\hat{\phi}_d\left(\beta_n,\frac{b}{2}+h_y\right) \tag{6a}$$

$$=G\left(\beta_n,\frac{b}{2}+h_y\right)\hat{f}(\beta_n) \tag{6b}$$

where $\hat{\phi}_c(\beta_n,b/2+h_y)$ and $\hat{\phi}_d(\beta_n,b/2+h_y)$ are the potential distribution functions in the transform domain, on the strip conductor and the complementary region, respectively, at the interface $y=b/2+h_y$. $G(\beta_n,b/2+h_y)$ is the Green's function at the plane $y=b/2+h_y$. Since $\hat{\phi}_d(\beta_n,b/2+h_y)$ and $\hat{f}(\beta_n)$ are

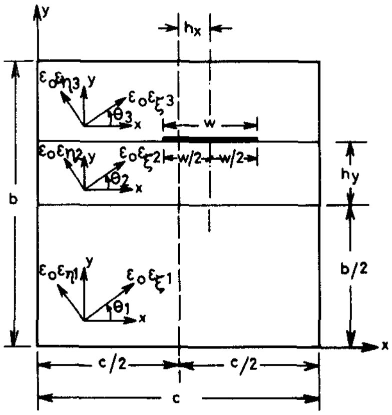

Fig. 1. Cross section of a rectangular coaxial line with offset inner conductor sandwiched between three anisotropic dielectrics.

nonzero over complementary regions, we can eliminate $\hat{\phi}_d(\beta_n, b/2+h_y)$ from (6) by using Parseval's theorem. Denoting the potential on the strip conductor $\phi_c(x, b/2+h_y)$ as V and noting that $V=Q/C$, where Q is the total charge on the strip conductor and C is the line capacitance per unit length, we get after some algebraic manipulations

$$\frac{1}{C}=\frac{\displaystyle\sum_{n=2,4,\cdots,\infty}\frac{4}{n\pi Y}\left[\int_{\text{strip}} f(x)\sin\beta_n x\,dx\right]^2}{\left[\displaystyle\int_{\text{strip}} f(x)\,dx\right]^2}. \tag{7}$$

The line capacitance of rectangular coaxial line can now be obtained by substituting an appropriate charge distribution $f(x)$ on the strip conductor in (7) and evaluating the integrals. Assuming a charge distribution on the strip conductor given by [6]

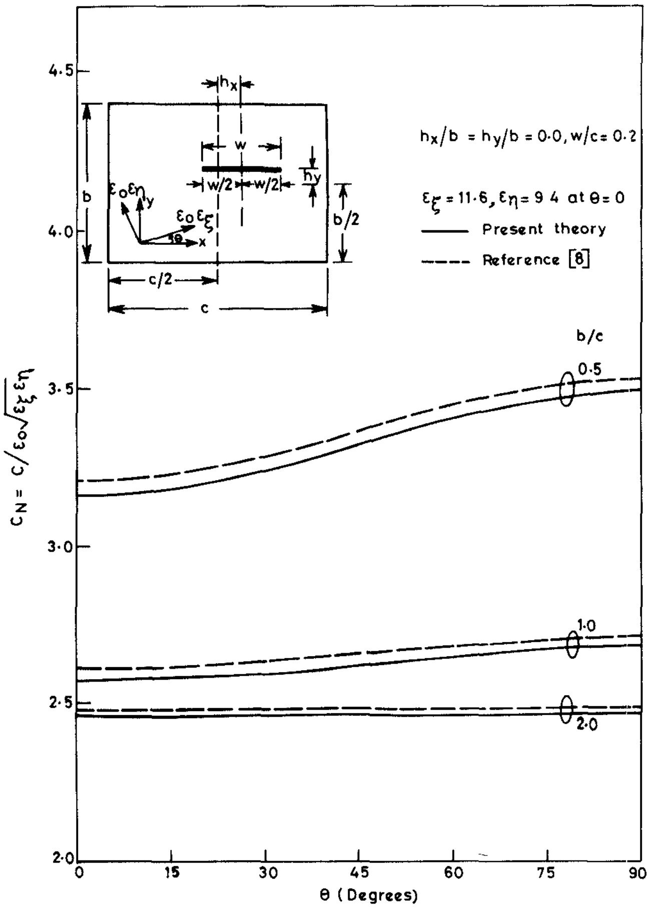

Fig. 2. Comparison of the normalized capacitance C_N of rectangular homogeneous coaxial line computed using present theory with those reported in [8].

$$f(x)=\frac{1}{w}\left[1+A\left|\frac{2}{w}\left(x-\frac{c}{2}-h_x\right)\right|^3\right],\quad \frac{c}{2}+h_x-\frac{w}{2}\leqslant x\leqslant\frac{c}{2}+h_x+\frac{w}{2}$$
$$=0,\quad \text{otherwise.} \tag{8}$$

The constant A in the charge distribution is obtained by maximizing the line capacitance C, i.e., by setting $\partial C/\partial A=0$. Strictly speaking, a symmetrical charge distribution given above is not the true representation of the charge distribution for the problem with an offset inner conductor ($h_x\neq 0$). However, since (7) is variational, a symmetrical distribution (8) will introduce only second-order errors in the capacitance calculation. For large h_x/b (very close to 0.50 $(c-w)/b$), the following nonsymmetrical distribution should yield more accurate results:

$$f(x)=\frac{1}{w}\left[1+A\left|\frac{1}{w}\left(x+\frac{w}{2}-\frac{c}{2}-h_x\right)\right|^3\right],\quad \frac{c}{2}+h_x-\frac{w}{2}\leqslant x\leqslant\frac{c}{2}+h_x+\frac{w}{2}$$
$$=0,\quad \text{otherwise.} \tag{9}$$

In this paper, all computations have been carried out using the charge distribution given by (8).

III. Numerical Results

Using the capacitance formula (7), numerical data on the capacitance of rectangular homogeneous/inhomogeneous coaxial line having an infinitesimally thin offset strip conductor embedded in multilayer anisotropic dielectrics have been generated. Since the capacitance formula (7) is variational in nature, the capacitance values obtained will always be less than the true value. As a numerical check, the normalized capacitance $C_N = C/\epsilon_0\sqrt{\epsilon_\xi\epsilon_\eta}$ of the rectangular coaxial line ($\epsilon_{\xi i}=\epsilon_\xi$, $\epsilon_{\eta i}=\epsilon_\eta$, $\theta_i=\theta$, $i=1,2,3$) have been computed for the zero offset case ($h_x=h_y=0.0$). These results are compared with those reported by Shibata *et al.* [8] in Table I. The dielectric substrate chosen is sapphire ($\epsilon_\xi=11.6$, $\epsilon_\eta=9.4$ at $\theta=0$). The comparison shows good agreement, the present results being slightly lower. The normalized capacitance C_N as a function of θ for various values of b/c, computed using present theory, are compared in Fig. 2 with the results reported in [8]. The comparison shows good agreement, and once again our results are slightly lower. The normalized capacitance C_N of a square homogeneous coaxial line filled with sapphire dielectric as a function of edge-offset h_x for various values of θ are plotted in Fig. 3. It is observed that C_N increases as θ or h_x/b increases.

The capacitance of rectangular inhomogeneous coaxial line is obtained by setting $\epsilon_{\xi 2}=\epsilon_\xi$, $\epsilon_{\eta 2}=\epsilon_\eta$, $\theta_2=\theta$ and $\epsilon_{\xi 1}=\epsilon_{\xi 3}=\epsilon_{\eta 1}$

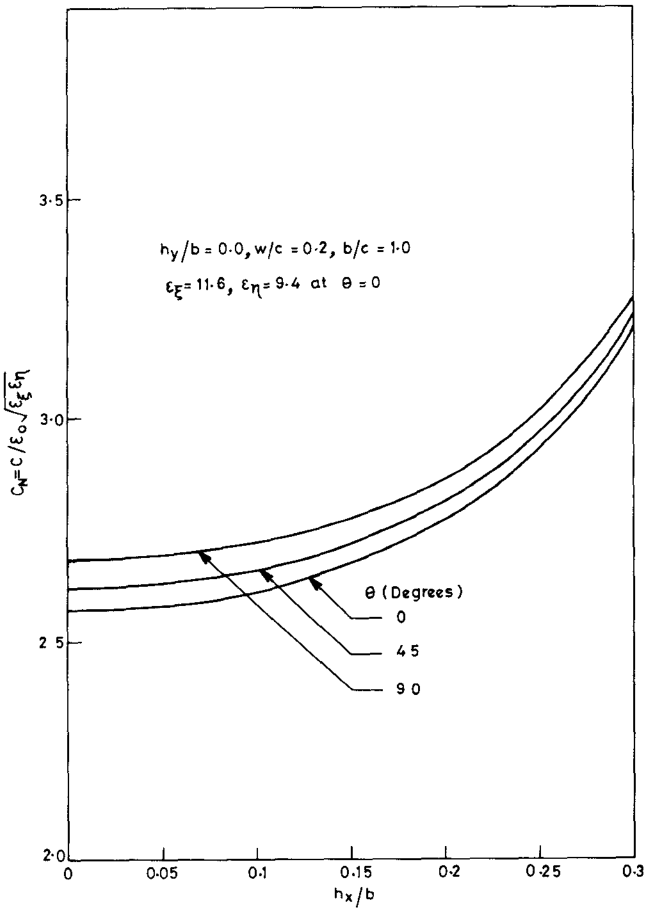

Fig. 3. Variation of the normalized capacitance C_N of square homogeneous coaxial line with sapphire dielectric as a function of edge-offset h_x/b with θ as the parameter.

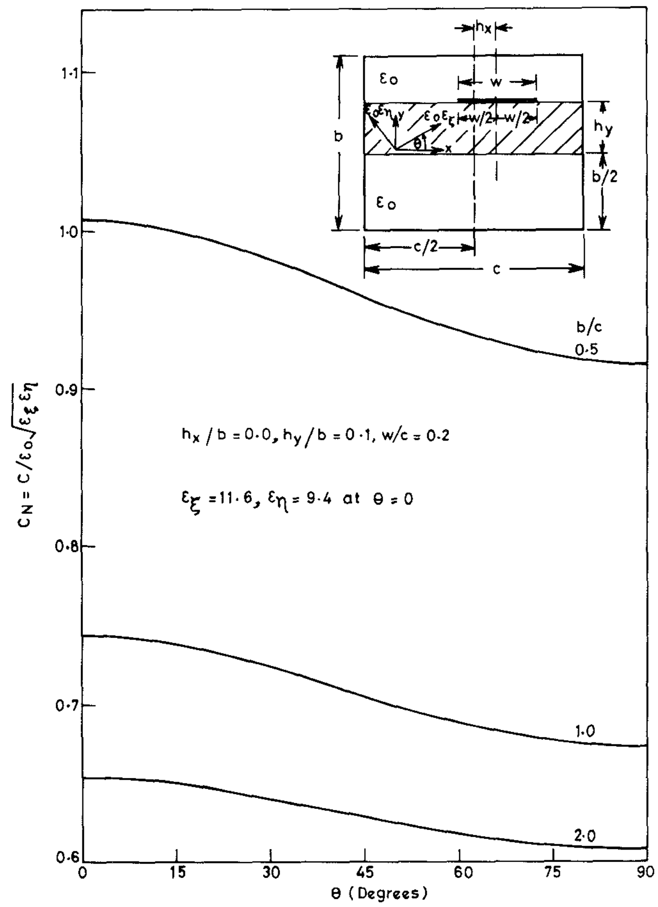

Fig. 4 Normalized capacitance C_N versus θ for the rectangular inhomogeneous coaxial line with b/c as the parameter.

TABLE I
COMPARISON OF THE CAPACITANCES $C/\epsilon_0\sqrt{\epsilon_\xi\epsilon_\eta}$ PER UNIT LENGTH FOR THE ZERO-OFFSET STRUCTURES WITH SAPPHIRE DIELECTRIC USING PRESENT THEORY WITH THOSE REPORTED BY SHIBATA *et al.* [8]

$h_x = 0.0$, $h_y = 0.0$					
b/c	w/c	θ	$C/\varepsilon_0\sqrt{\varepsilon_\xi\varepsilon_\eta}$		
			Shibata et al [8]		Present theory
			Method B	Method A	
0·5	0·1	0	2·37875	2·37875	2·359050
		90	2·56979	2.56979	2·546988
1·0	0·2	0	2·59301	2·59301	2·569020
		90	2·70460	2·70460	2·678785
2·0	0·6	0	4·63194	4·63198	4·539965
		90	4·65335	4·65329	4·561012

$= \epsilon_{\eta 3} = 1$, $\theta_1 = \theta_3 = 0$ in (7). For a fixed value of h_y/b and w/c, the variations of C_N are plotted as a function of θ for three different values of b/c in Fig. 4. The substrate chosen is sapphire. For a fixed value of θ, C_N decreases as b/c is increased. This variation is similar to that obtained for rectangular homogeneous coaxial lines (Fig. 2). For a fixed value of $b/c, C_N$ decreases as θ increases. This trend is opposite of that obtained in the case of rectangular homogeneous coaxial lines. The variation of C_N as a function of edge-offset h_x for various values of θ is plotted for square inhomogeneous coaxial lines in Fig. 5. The variations obtained for a fixed value of θ are similar to those obtained in the case of square homogeneous coaxial lines (Fig. 3). On the other hand, for a fixed value of $h_x/b, C_N$ decreases as θ is increased. This trend is opposite of that obtained in the case of square homogeneous coaxial lines.

IV. CONCLUSIONS

A simple analytical technique for analyzing capacitance characteristics of rectangular homogeneous/inhomogeneous coaxial lines with an offset inner conductor having anisotropic dielectrics is presented. Numerical data are presented on the capacitance of rectangular and square coaxial lines. The effects of varying the angle of tilt θ and offsetting the strip conductor horizontally as well as vertically are studied.

The numerical data presented in this paper are quite accurate for most practical applications. The accuracy can further be increased by assuming a more complex charge distribution in the form of a polynomial on the strip conductor. This will, however, increase the computational time.

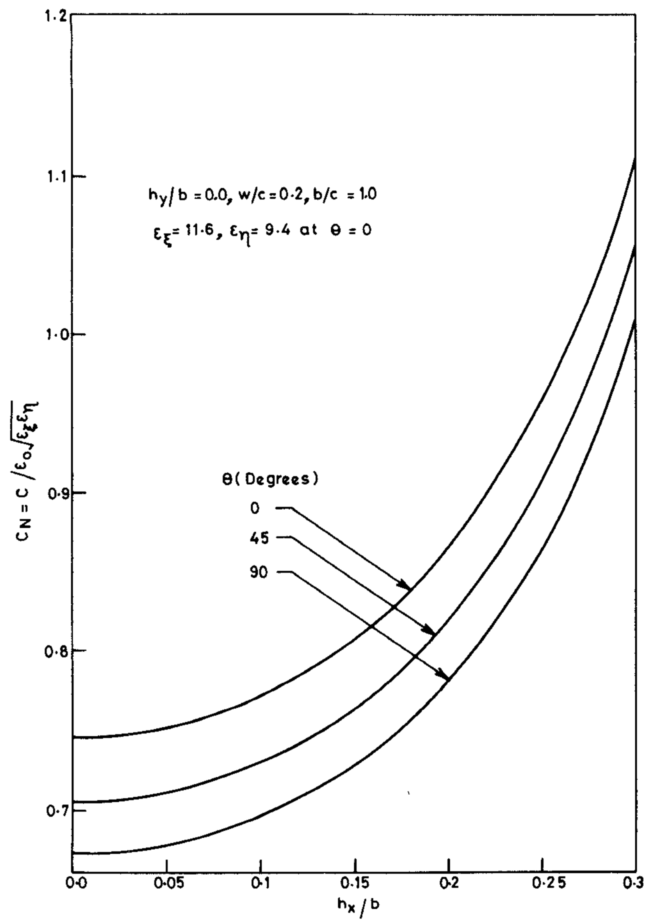

Fig. 5. Variation of the normalized capacitance C_N of square inhomogeneous coaxial line with sapphire dielectric as a function of edge-offset h_x/b with θ as the parameter.

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Incremental Frequency Rule for Computing the Q-Factor of a Shielded TE_{omp} Dielectric Resonator

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Abstract—**The principle of Wheeler's incremental inductance rule is applied to the TE_{omp} cylindrical resonator within a metal enclosure. The procedure permits one to compute the conductor losses solely from the decrease in resonant frequency when the metal walls are receded for one skin depth.**

I. INTRODUCTION

Computation of the Q-factor of metal cavities requires an integration of the dissipated power over the entire metal surface of the cavity. When a dielectric resonator is placed within the metal enclosure, analytical expressions for the field distribution become quite involved, and the numerical evaluation of Q-factor frequently requires various simplifying assumptions in order to render the solution possible [1], [2].

It is well known that the computation of conductor losses on the TEM transmission lines may be considerably simplified by using the "incremental inductance rule" developed by Wheeler [3]. This rule replaces the detailed surface integration by a simple computation of the increment in inductance per unit length when all the metal walls are receded by $\delta/2$, where the skin depth δ is given by

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}. \qquad (1)$$

In the above, f is the frequency of operation, σ is the conductivity, and μ is the permeability of the metal walls.

It will be shown here that a similar trick can be applied also to the TE_{omp} modes in rotationally symmetric hollow resonators, but the increment which is to be calculated is now the increment in the resonant frequency.

II. THE RULE

The Q-factor due to conductor losses of any cavity consisting of a rotationally symmetric metal enclosure, supporting the TE_{omp}-type field, can be computed as follows:

$$Q_c = \frac{f_0}{\Delta f_0(\delta)}. \qquad (2)$$

In the above, f_0 is the resonant frequency of the cavity, computed for the case when the metal enclosure is made of a perfect conductors. $\Delta f_0(\delta)$ is the increment in the resonant frequency, computed again for perfectly conducting walls which are now moved inwards for one full skin depth δ, evaluated by (1).

III. PROOF

Fig. 1 depicts a cylindrical dielectric resonator within a metal enclosure. When the enclosure is made of a perfect conductor, the knowledge of the magnetic-field intensity as function of position permits one to calculate the total stored magnetic energy W_m. When the enclosure is made of a conductor with finite conductiv-

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